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# ON THE USE OF A HYBRID BUBBLE CHAMBER IN THE 100-BeV REGION

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## ABSTRACT

An analysis has been made of 100-BeV interactions in a bubble chamber hybrid spectrometer. The following conclusions may be drawn:

1. High-efficiency, high-precision $\gamma$ detection is a necessity since very poor precision is attained in the invariant missing mass from momentum measurements alone.
2. The criteria of $\delta P = 0.1$ and $P\delta\theta = 0.1$ GeV/c precision measurement are not very appropriate for a broad spectrum instrument. A more reasonable set would be $\delta P = 0.2$ and $P\delta\theta = 0.02$ or $0.01$ GeV/c.

The reason for the above can be understood by looking at the reactions in the overall center-of-mass. The forward-going tracks would characteristically be measured with a precision of $\pm 7$ MeV/c whereas the backward going nucleon would be measured with a precision of 50-70 MeV/c with the errors coming from multiple scattering. Angular errors produce errors mainly in transverse momentum (for fast tracks) and are invariant to Lorentz transformation. Thus errors of no more than 10-20 MeV/c can be tolerated for $P\delta\theta$.

This report is written in response to proposals for a large bubble chamber and a hybrid system of chamber plus spectrometer as advocated by Fields et al. and myself.

## LIMITATIONS IN THE USE OF THE BUBBLE CHAMBER

### Scanning

It is difficult to scan efficiently for interactions in a bubble chamber at very high energy by a simple area search. The high degree of collimation and small angles of deflection make this the case for interactions with low multiplicities. It is probably necessary to have some sort of external indication as to where the interaction occurred. (For example: display by means of a hodoscope which beam track has interacted.) It is extremely important to allow no more than one interaction in the chamber or immediately upstream since crossing tracks are very difficult to scan and measure through. This fact plus the importance of $\gamma$-ray detection without confusion makes it important to have a considerable magnetic deflection immediately upstream of the chamber.

A particular difficulty with a chamber has to do with the short recoils at high energy. At 25 BeV in the reaction $\pi^- + p \rightarrow \pi^- \pi^+ \pi^- p$ the order of 5 - 10% of the events were missed in scanning because of the shortness of the recoiling proton. This inefficiency could well increase to 20-30% at 100 BeV. As will be pointed out later the problems produced by multiple scattering and short range is a fundamental limitation of bubble chambers in the 100-BeV region. At high energies precision in angle measurements becomes increasingly important. It is probably necessary to increase the angular precision obtainable with currently used systems. (Lower f stops of lenses for example.)

## KINEMATIC CONSIDERATIONS

In order to proceed further it is necessary to look at the kinematics of a few simple reactions. Let's consider $\pi^- + p \rightarrow X^- + p$ where $X^-$ might be any object of mass up to 4 BeV/$c^2$. When the mass is increased above the $\pi$ a minimum longitudinal momentum is imparted to the nucleon as shown in Fig. 1. Figure 2 gives a graph of the mass produced versus $\Delta_{min}$.

$$\Delta_{min} = \frac{m^{*2} - m_\pi^2}{2P} .$$

Suppose we wish to determine the mass of $m^*$ by using the recoiling proton as in a missing mass spectrometer. In order to do this we must look at both the energy and angle of the recoiling proton. A limited kinematic plot for the recoiling proton is given in Fig. 3 in which we plot transverse momentum versus longitudinal momentum of the proton for different mass objects $X^-$. Approximate kinematics are relatively simple to generate. To add to the plot for a given value of transverse momentum ($P_T$) one can from the elastic kinematics obtain $X^- + P$ by simply adding $\Delta_{min}$ to the $P_{Long}$ for the elastic case.

Let's consider the problem of discriminating between $\rho^- - p$ and $A^-p$ for a range of transverse momentum. Consider first $P_T$ = 200 MeV/c:

$$\Delta_\rho = 3\ \mathrm{MeV/c},$$

$$\Delta_A = 8\ \mathrm{MeV/c}.$$

The multiple scattering of the proton produces an error angle of the order of

$$\delta\theta = \frac{13}{P\beta}\sqrt{\frac{\ell}{800}}\ ,$$

$$P\delta\theta = \frac{13}{\beta}\sqrt{\frac{\ell}{800}} = \frac{13}{0.15} \times \frac{1}{20} = 4.3\ \mathrm{MeV/c},$$

= Uncertainty in longitudinal momentum due to multiple scattering

$(P\delta\theta)_{set}$ = 2 MeV/c = setting error

= Uncertainty because of error in setting which is important in the case of a stopping track.

Next let us consider the case of $P_T$ = 400 MeV/c. The same calculations give

$$P\delta\theta = 3\ \mathrm{MeV/c}.$$

There is in addition an error coming from uncertainty in the length of the proton momentum vector. This produces an additional uncertainty in longitudinal momentum of about 3 MeV/c. This gives a total uncertainty of the order of 5 MeV/c. On the curves we have put the range of errors to be expected on the longitudinal momentum. At low momenta one can measure the momentum of the proton by range which produces a precision of 1-2% so that the error in longitudinal momentum is produced mainly by uncertainty in angle produced by multiple scattering and to a lesser extent by setting errors. As the transverse momentum is increased the longitudinal momentum in the elastic and inelastic processes increases quadratically.

The error in momentum determination produced by multiple scattering is characteristically 4-5% in the range 300-800 MeV/c. This means an error in longitudinal momentum of the order of 10 MeV/c at 600 MeV/c. The relative error produced in multiple scattering decreases like $1/\beta$ as the momentum of the nucleon is increased. This still produces an error in longitudinal momentum of the order of 10 MeV/c. Since we need to measure the longitudinal momentum with a precision of typically 5 MeV/c the precision in missing mass determination decreases.

The conclusion to be drawn from these considerations is that the recoiling proton gives very poor precision in determining the missing mass. It does, however, give a good measure of the transverse momentum of the missing mass.

Next let us consider the system $X^-$. We are specifically considering the case in which we see a single charged particle which might be an example of the reaction

$$\pi + P \rightarrow P + X^-$$
$$\hookrightarrow \pi^- \pi^0, \text{ or } \pi^- \pi^0 \pi^0.$$

What is observed directly is the momentum of the negative track.

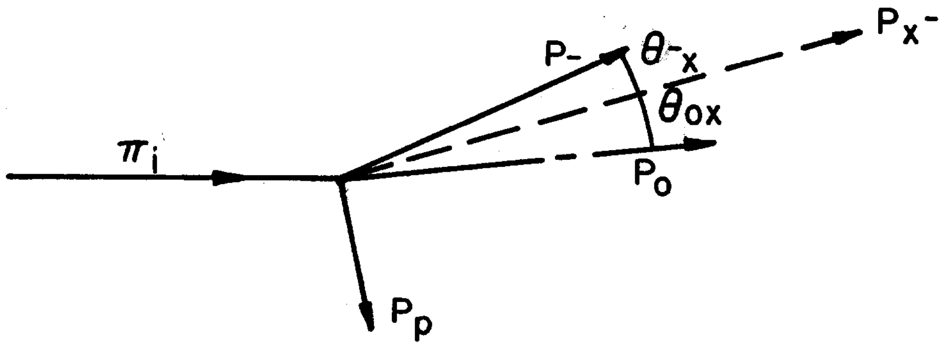


Fig. 4.

In the above diagram $P_-$ and $P_p$ are measured directly. The uncertainty in the longitudinal component of $P_p$ produces an uncertainty in the mass of $X^-$ of 1 to 2 BeV/$c^2$. The direction of $P_{x^-}$ is with high precision that of an elastically scattered $\pi^-$ for that value of $P_T$. Let $\theta_{-x}$ be the angle between $P_-$ and $P_{x-}$. $P_o$ the momentum of the neutral can obviously be constructed. The invariant mass of $X^-$ is

$$M_{x-}^2 = (E_- + E_o)^2 - (P_- + P_o)^2$$

$$= m_-^2 + m_o^2 + 2E_-E_o - 2P_-P_o \ .$$

Making relativistic approximations,

$$E_- = P_- + \frac{m_-^2}{2P_-}\ , \quad E_o = P_o + \frac{m_o^2}{2P_o}\ ,$$

$$M_{x^-}^2 = m_-^2 + m_o^2 + 2P_- P_o \,(1 - \cos\theta_{-o}),$$

$$+ \frac{m_-^2 P_o}{P_-} + \frac{m_- 2P_o}{P_-} + \frac{m_o^2 P_-}{P_o} \ .$$

The last two terms can usually be dropped unless one of the momenta is low. A lower limit on the mass of $X^-$ can be set by assuming the negative and neutrals to be $\pi$'s. Under those circumstances $(M_{x^-})^2_{min} \simeq 2m_\pi^2 + P_- P_o \theta^2_{-o}$. Characteristically $P_- \theta_{-o} = 0.25$ (true in the $\rho$, f, g regions), then transposing them gives

$$m_o^2 = M_{x^-}^2 - m_-^2 - P_- P_o \theta^2_{-o}.$$

Making an equal angle approximation gives:

$$\theta_{-x} = \theta_{ox} = 1/2\ \theta_{-o}\ .$$

Then $P_- P_o \theta^2_{-o} \simeq 0.25$. Thus on the average:

$$m_o^2 = M_{x^-}^2 - m_-^2 - 0.25\ .$$

The uncertainty in $M_{x-}$ is the order of 1 to 2 BeV/$c^2$ because of the uncertainties in $P_{Long}$ at the target vertex. Thus

$$\delta m_o^2 \sim 2\,(\mathrm{BeV}/c)^2.$$

Thus with no $\gamma$ detection in the system the hybrid system is hopeless for this sort of reaction.

A similar analysis can be made of the reactions

$$\pi^- + p \rightarrow X^- + N^{*+}.$$

In such reactions we suppose that the $N^{*+}$ is slow in the lab. Such a reaction characteristically goes with $t = 0$ i.e. $K = K_o$ where $K$ is the momentum-energy transferred to the nucleon. The kinematics can again be generated from the elastic kinematics and give the $N^{*+}$ system recoiling with a momentum

$$P_{N^*} = P_{-T} + (\Delta_{x-} + K + P_L)\, e_{\parallel} \;.$$

($e_{\parallel}$ = unit vector in beam direction)

If the $N^*$ has a mass of 1.4 BeV/$c^2$ then $K$ would be 0.5.

$$\frac{K = M_{N^*}^2 - M_N^2}{2M_N}$$

This additional longitudinal momentum would make high precision in missing mass determination even more difficult than before. We are still seeking to measure $\Delta_{x^-}$ with a precision of 5-10 MeV/c which is impossible in a bubble chamber if we must measure a proton of 300-1000 MeV/c with a large component of longitudinal momentum.

## SOLUTION

This difficult situation can be resolved by means of high precision and efficient $\gamma$ detection. The previous analysis shows that this is not a frill but a necessity. The vector $P_o$ is well determined if $\delta P_o \approx 0.12$ and $P_o \delta\theta \approx 0.02$. The angular information can be improved from the $\gamma$ information. If you detect both $\gamma$'s from a $\pi^o$ and know with some reliability that there is only 1 $\pi^o$ then one can rely on a fitting program and the mass of $X^-$ can be determined with a precision which is determined mainly by angle errors,

$$M_{x-}^2 = m_o^2 + m_-^2 + P_- P_o \theta_{o-}^2 \theta^+$$

$$\delta M_{x-}^2 = \delta P_- P_o \theta^2_{-o} + \delta P_o P_- \theta^2_{o-}$$

$$+ 2\theta_{o-} P_- P_o \delta\theta_{o-}$$

$$\simeq \underset{\delta P_{\text{error}}}{\underset{\uparrow}{4 \times 10^{-5}}} + \underset{\delta\theta_{\text{error}}}{\underset{\uparrow}{10^{-2}}}$$

$$\overline{\delta M}_{x-} \simeq 100 \text{ MeV}/c^2$$

This simple analysis says that more effort should be made in angle measurement and less in longitudinal momentum measurement of the very high energy particles. A $\delta P$ of 0.2 at 100 BeV/c would be quite acceptable. There is a trade-off between $\gamma$ detection and momentum measurement. The $\gamma$ detection allows one to use fitting programs with confidence. This cannot be done otherwise.

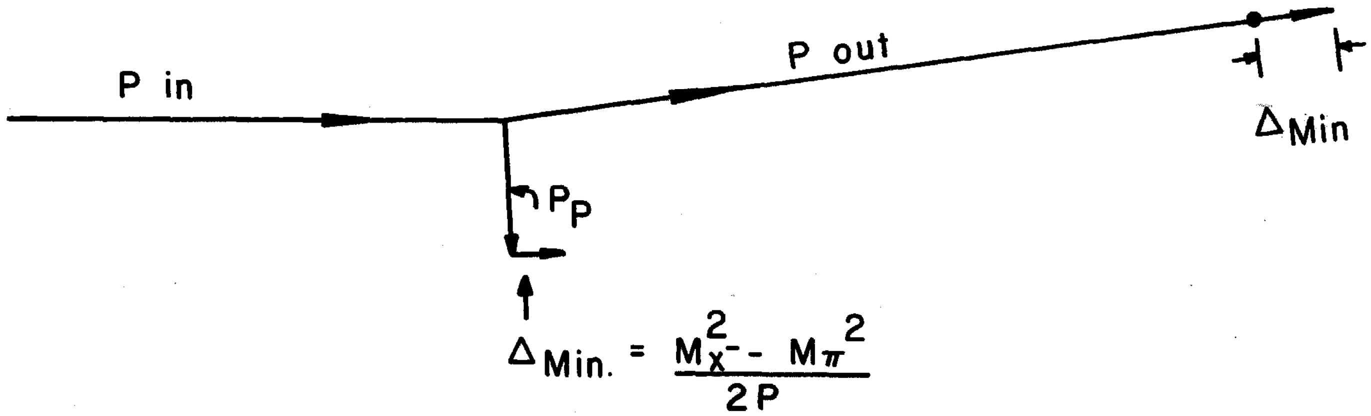


Fig. 1. Minimum longitudinal momentum imparted to recoil proton.

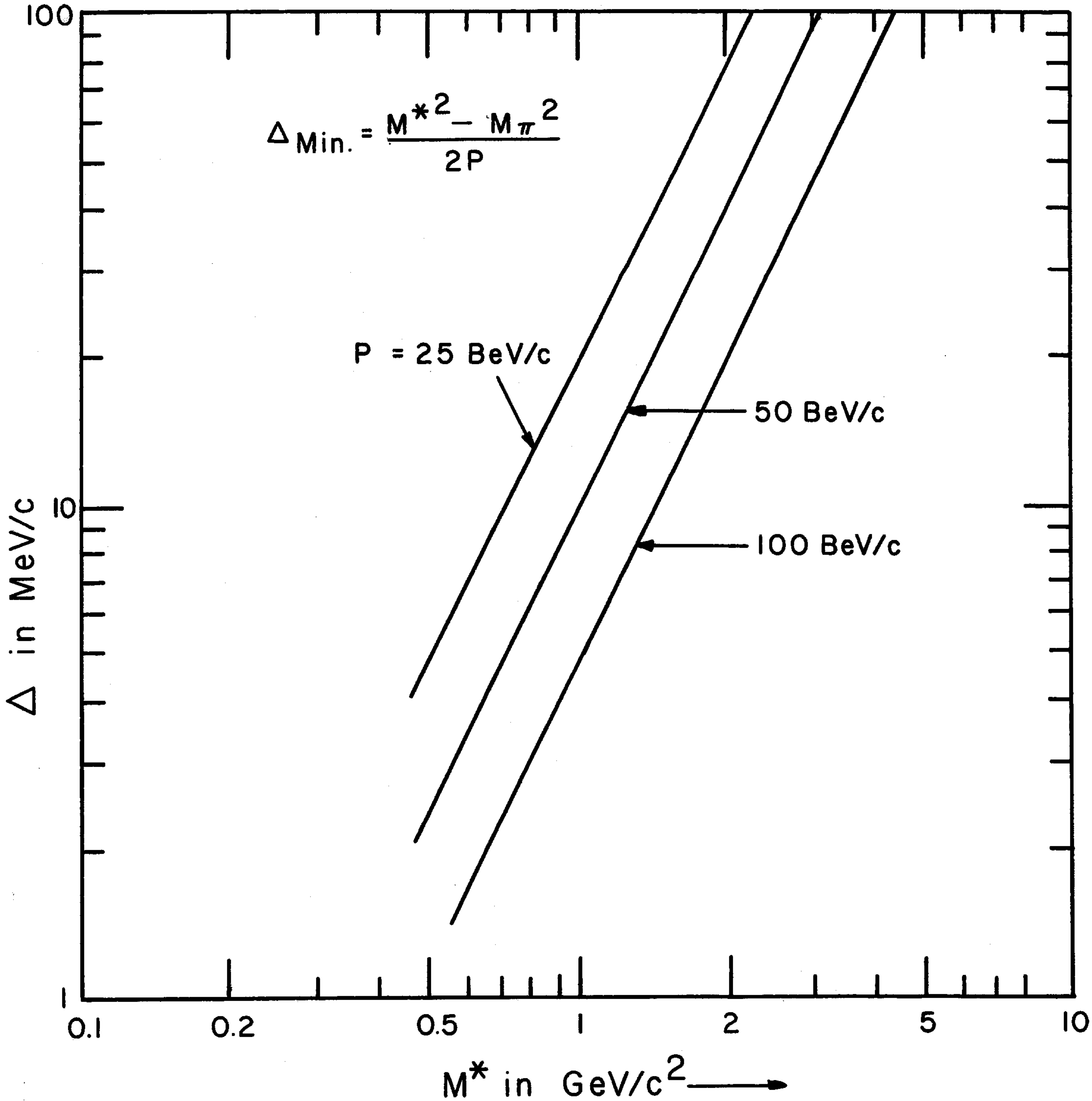


Fig. 2. Mass of $M^*$ produced vs minimum longitudinal momentum of nucleon for several interaction momenta.

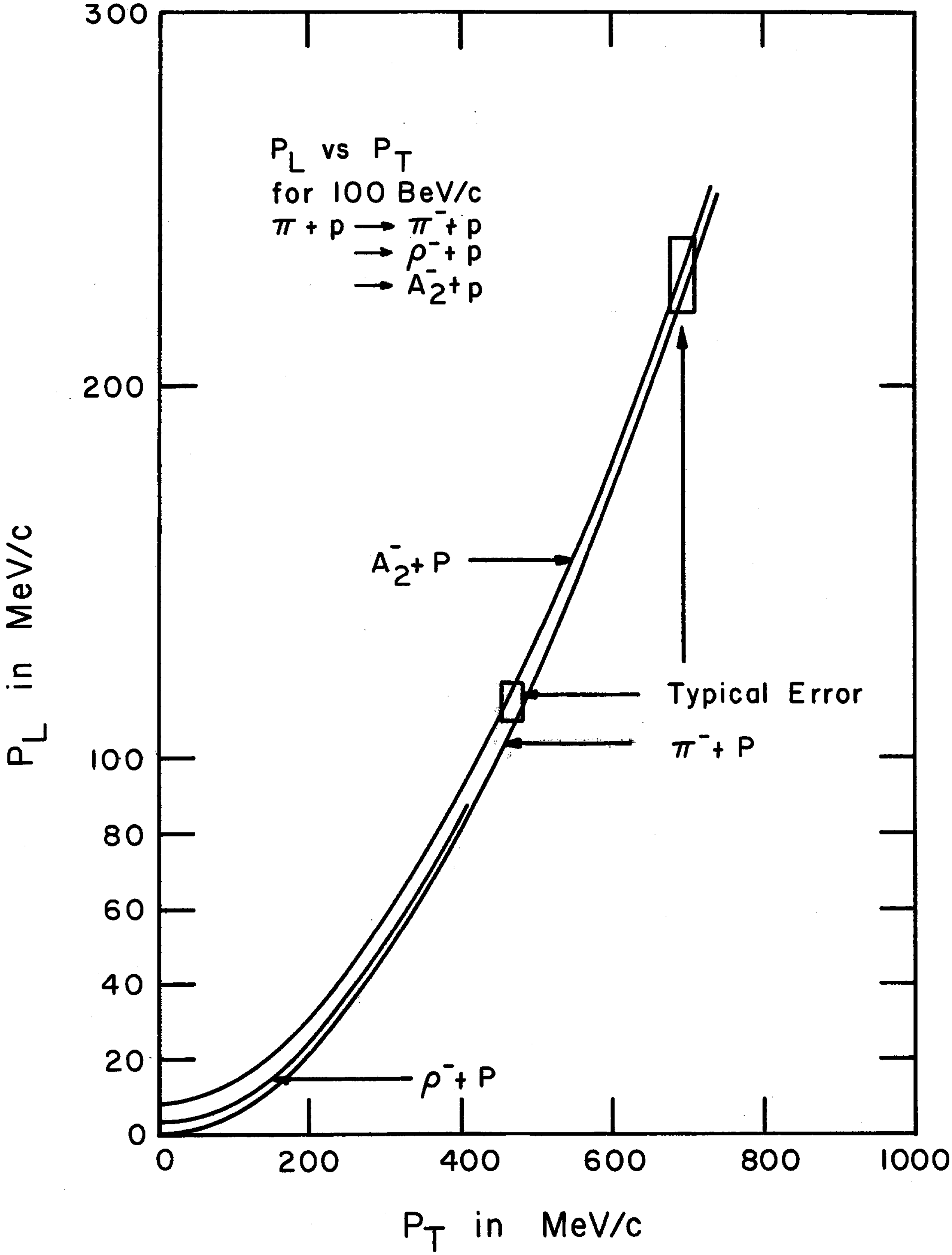


Fig. 3. Longitudinal vs transverse momentum of the recoil proton showing how poorly it discriminates different masses of $X^-$.